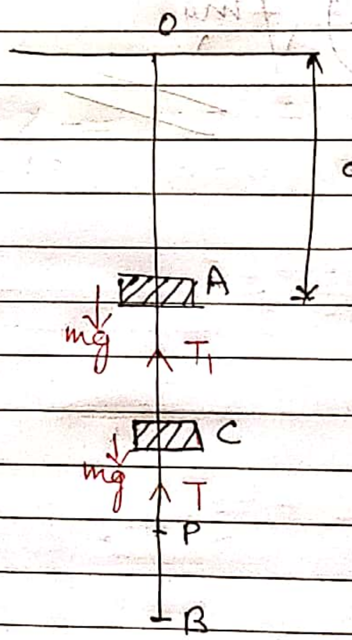
VERTICAL ELASTIC STRING:

Theorem:

— A light elastic string of natural length 'a' and modulus of elasticity λ is suspended by one end, and the other end is tied to a particle of weight mg, the particle is slightly pulled down and released. To discuss the motion of the particle.

Proof:–



Let $OA = a$ be the natural length of the string whose end 'O' is fixed and a particle of mass m is hanged on the other end A.

Suppose the string is extended upto C at which T_1 be the tension of the string which will balance the weight mg.

i.e. $T_1 = mg$

But, by Hooke's law

$$T_1 = \lambda \cdot \frac{AC}{a}$$

$$\Rightarrow mg = \lambda \cdot \frac{AC}{a}$$

$$\Rightarrow AC = \frac{mg \cdot a}{\lambda} \quad \text{—— (1)}$$

Now, the particle is pulled down to B and released then suppose P be the position of the particle at any time 't' of its motion such that

$AP = x$

Let T be the tension of string at P then by Hooke's law

$$T = \lambda \cdot \frac{x}{a} \quad \text{—— (2)}$$

Since the motion of the particle is towards A and therefore the equation of the motion

$$m\frac{d^2x}{dt^2} = -(T - mg)$$

$$m\frac{d^2x}{dt^2} = -\left(\frac{\lambda x}{a} - mg\right)$$

$$m\frac{d^2x}{dt^2} = -\frac{\lambda}{a}\left(x - \frac{mga}{\lambda}\right)$$

$$\Rightarrow \frac{d^2x}{dt^2} = -\frac{\lambda}{ma}(x - AC)$$

$$\frac{d^2x}{dt^2} = -\mu(x - AC) \quad (3)$$

where, $\mu = \frac{\lambda}{ma}$

put $x - AC = z$

$$\Rightarrow \frac{dx}{dt} = \frac{dz}{dt}$$

$$\Rightarrow \frac{d^2x}{dt^2} = \frac{d^2z}{dt^2}$$

$$(3) \Rightarrow \frac{d^2x}{dt^2} = -\mu z \quad (4)$$

which shows that the motion of the particle is a S.H.M. with time period / $\frac{2\pi}{\sqrt{\mu}}$

$$= \frac{2\pi}{\sqrt{\frac{\lambda}{ma}}}$$

$$= 2\pi\sqrt{\frac{ma}{\lambda}}.$$